

Notes on English grammar

Andrew McKee

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Some notes from *Scholarly English* discussing common grammar issues in English speaking and writing. Rather than trying to give an exhaustive list of all rules these notes highlight the most common mistakes; the aim is to make students more aware of these mistakes, and to help develop some intuition for what sounds correct to native speakers.

Articles

Formally, an *article* is a part of language/speech which is used together with a noun (or noun phrase) to indicate identifiability of the noun; that is, the article indicates what we know, or are expected to know, about the noun.

Not every language uses articles, but they are very important in communicating mathematics in English.

The most common articles in English are *a*, *an* and *the*.

Exercises and examples

What is wrong with the following sentences?¹

- (1) The function $-e^{-x}$ is derivative of e^{-x} . The function $-e^{-x}$ is a derivative of e^{-x} .
- (2) Let A be the matrix with determinant 0.
- (3) This is common in the Graph Theory.
- (4) Multiplying the equation (1) by 3 gives the result.
- (5) The properties of the groups.

Put the correct article (if required) in the blank space.

- (a) This is ____ example of a system of equations with infinitely many solutions.
- (b) We use ____ Theorem 2.4.

¹The examples are taken from *Writing Mathematical Papers — a Few Tips*, Trzeciak, available at <https://www.impan.pl/en/publishing-house/for-authors>.

- (c) ____ intersection of ____ sets A_i .
- (d) ____ number $3 + 2i$ is ____ complex number.
- (e) He is ____ mathematician. They are ____ mathematicians.
- (f) ____ real number satisfies the equation $x^2 = -1$.

Rules for articles

Here are some rules for using articles.² There are more rules, and some exceptions, but the ones below are the most important for mathematical writing.

1. Use *the* (the definite article) when the listener/reader is thought or expected to know exactly what is being referred to. This is most commonly one of the following situations:
 - (a) speaking about something uniquely specified (see question (1) above);
 - (b) speaking about something which has already been mentioned (see question (c) above).
2. Use *a* (indefinite article) when speaking about something not uniquely specified (exercises (2) and (d)).
3. Use *an* (indefinite article) when *a* is correct, but the noun begins with a vowel sound (example (a)). Be careful: “a university” is correct because in this case the ‘u’ of university is not pronounced with a vowel sound, unlike “umbrella”, where the ‘u’ has a vowel sound, hence “an umbrella” is correct. “An hour” is also correct even though ‘h’ is not a vowel; depending on your accent it may be “a hotel” or “an ’otel”. We use *an* in place of *a* to avoid having to pronounce two vowel sounds in a row.
4. Use no article in the following cases:
 - (a) when the noun is a name (example (b)); there are exceptions to this rule, for example “the United States of America” is correct, but the exceptions are not very common in mathematics;
 - (b) when the noun refers to a body of theory (example (3), another example is “in classical analysis”);
 - (c) often when the noun is plural, or a mass noun (example (e));
 - (d) when writing an instruction articles are often omitted, for example if describing a mathematical algorithm;
 - (e) (there are other examples which are not so important for us).

²These rules are taken from the British Council’s *LearnEnglish* website, <https://learnenglish.britishcouncil.org/grammar/english-grammar-reference> and the Wikipedia article [https://en.wikipedia.org/wiki/Article_\(grammar\)](https://en.wikipedia.org/wiki/Article_(grammar)).

5. The word *no* can be used as an article, indicating none of its noun (example (f)).
6. The word *some* can also function like an article. In mathematics one sees things like “ $\{f(x) : x \in \mathbb{R}\}$ is bounded by some constant C ”, where *some* could be replaced by a ; usually in English it is used to indicate a non-specific quantity, for example “some water”.

With these rules in mind, articles in mathematical English often signal some context to the reader/listener. It is these signals which students get wrong, even when they are native English speakers, and this has a negative effect on the quality of their communication.

Try to explain the difference in meaning between the two sentences in each case.

- (i) Take a union $A \cup B$. Take the union $A \cup B$.
- (ii) The equations always have solutions. Equations always have solutions.
- (iii) Let $p_1^{n_1} p_2^{n_2} \cdots p_m^{n_m}$ be a prime factorisation of the natural number k . Let $p_1^{n_1} p_2^{n_2} \cdots p_m^{n_m}$ be the prime factorisation of the natural number k .

Answers and discussion

- (1) The function $-e^{-x}$ is *the* derivative of e^{-x} . (The derivative is unique. Rule 1a.)
- (2) Let A be *a* matrix with determinant 0. (There are many matrices with determinant 0. Rule 2.)
- (3) This is common in Graph Theory. (Graph Theory is a body of work, so no article is used. Rule 4b.)
- (4) Multiplying equation (1) by 3 gives the result. (Because equation (1) has a name we do not use an article. Rule 4a. If the equation has no name we would use *the*, for example “multiplying the equation above by 3 gives the result”.)
- (5) The properties of groups. (When referring to groups in general no article is used. Rule 4c. If some specific groups were being discussed then *the* is used, as in “the properties of the groups \mathbb{Z} and \mathbb{Z}_n ($n \in \mathbb{N}$).)
 - (a) This is *an* example of a system of equations with infinitely many solutions. (There are many systems of equations with infinitely many solutions, this one is not special, so we use an indefinite article; it is “an” rather than “a” because “example” begins with a vowel sound. Rule 3.)
 - (b) We use Theorem 2.4. (No article because the theorem is named. Rule 4a.)
 - (c) *The* intersection of *the* sets A_i . (The intersection is uniquely specified; the sets A_i are already known to the reader. Rule 1.)

- (d) *The* number $3 + 2i$ is *a* complex number. (The number $3 + 2i$ is uniquely specified, Rule 1. The *a* indicates it is one of many complex numbers and not otherwise special, Rule 2. If this number is important in context *the* may be correct, such as “the number $3 + 2i$ is *the* complex number we are looking for”.)
- (e) He is *a* mathematician. They are mathematicians. (The indefinite article indicates there is nothing special about the person among mathematicians, or that there is no more information, Rule 2. Plurals do not usually take articles, Rule 4c.)
- (f) *No* real number satisfies the equation $x^2 = -1$. (Here *no* functions as a *negative article*, meaning none of the noun. Rule 5.)
- (i) In the first sentence the “a” tells the reader that there is nothing special about the union $A \cup B$; likely something will be said that applies to all unions. In the second sentence “the” indicates that the reader should already know what A and B are.
- (ii) The first sentence indicates that the reader should already know what equations are being discussed, while the second sentence is a statement about all equations (here “all” could be restricted by some other context).
- (iii) In this simple example using “a” makes us suspect the writer does not understand correctly, since we know that prime decompositions are unique. In a more complicated example making this mistake could be very confusing for a reader who does not already understand the material.

Agreements

In language *agreements* are when a word changes form according to the other words to which it relates. English features agreements based on *person* and *number*; other languages have agreements on *gender* and/or *grammatical case*, though these rarely occur in English.

Mistakes with agreements can make mathematical statements ambiguous, so avoiding these mistakes is a useful part of being a good communicator of mathematics. The goal here is to give some examples to make students aware of these mistakes and build intuition for correct agreements.

Person

Formally, *grammatical person* is what distinguishes the speaker/writer (first person, *I am*), the addressee (second person, *you are*) and others (third person, *he/she/it is*). Mistakes with this type of agreement are unlikely to cause confusion, but correcting them does make speaking sound more natural.

Find and correct the mistakes in each sentence below.

- (1) It mean that the group is amenable.
- (2) I tell that the operation is commutative. It telling that the operation is commutative.
- (3) One see that the determinant is 0.
- (4) You see that the determinant is 0.

Number

The *grammatical number* in English concerns how words change to indicate one (*singular*) or many (*plural*) of something being discussed. In mathematics it is often very important if there are one or many of a certain object, so mistakes can lead to ambiguity.

Find and correct the mistakes in each sentence below.

- (a) There are unique prime factorisation of every natural number.
- (b) This examples are the most common ones.
- (c) Some papers are better than others.
- (d) Mathematics are the study of numbers, axiomatic systems, ...

Solutions

- (1) It *means* that the group is amenable. (It would be correct to say *I mean that*, but in the third person *means* is used.)
- (2) *I tell* is correct grammar, but this phrasing sounds very unnatural in English; *I am saying that...*, or *I simply tell you that...*, or *I claim that...* could all be correct, depending on context (there are many other possibilities). *It tells us that...* or *it is telling us that...* are both correct (note that including *us* makes the English sound more natural).
- (3) One *sees* that the determinant is 0. (In third person singular this is the correct agreement; note that *they see* is correct no matter if *they* is singular or plural.)
- (4) This one does not need any corrections.
- (a) There *is a* unique prime factorisation of every natural number. (Singular means we use *is*. Note the indefinite article *a* — anything else sounds unnatural here, even though it does not seem to match with the rules for articles.)
- (b) This *example is* the most common *one*, or *These* examples are the most common ones. (Note the ambiguity: we cannot tell from the original sentence which part is the mistake, which could cause confusion.)

- (c) This one is correct if *paper* means a unit of writing, such as a research article or an exam, rather than the material; if the material meaning is intended then one needs to say *Some paper is better than other paper*, or similar.
- (d) Mathematics *is* the study... (Even though it looks plural, *mathematics* is a mass noun (also called an uncountable noun); mass nouns have no concept of singular or plural, but grammatically they work like single objects.)